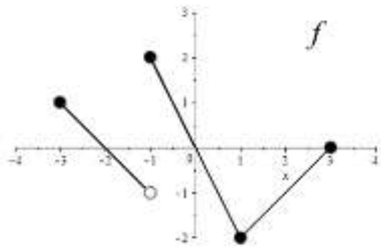
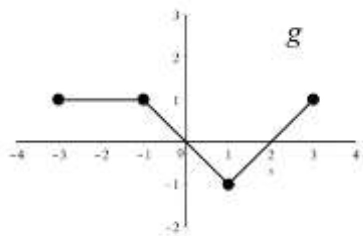
New Limits from Old
Homework Assignment

Use correct limit notation and words, where appropriate, to express your answers to the following problems.

1. Let f and g be the functions whose graphs are shown below. Use the graphs to evaluate the following limits. If a limit doesn't exist, explain why.



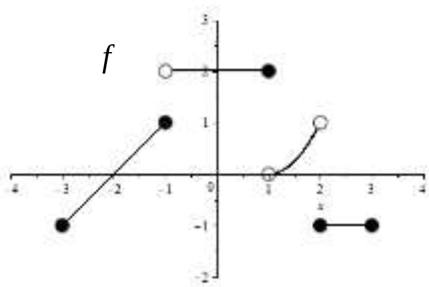


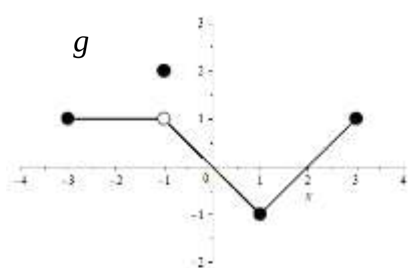
a. $\lim_{x\to -3^-} \frac{f(x)}{g(x)}$

b. $\lim_{x\to 0} \frac{f(x)}{g(x)}$

c. $\lim_{x\to 1} \frac{f(x)}{g(x)}$

2. Let f and g be the functions whose graphs are shown below. Use the graphs to evaluate the following limits. If a limit doesn't exist, explain why.





a. $\lim_{x\to 0}\left(f(x)+g(x)\right)$

b. $\lim_{x\to 2}\left(f(x)+g(x)\right)$

c. $\lim_{x\to 1}\left(f(x)g(x)\right)$

d. $\lim_{x\to 2}\left(f(x)g(x)\right)$

e. $\lim_{x\to -2} \frac{f(x)}{g(x)}$

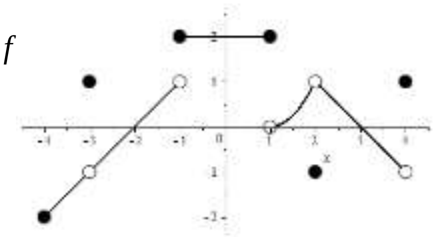
f. $\lim_{x\to 0} \frac{g(x)}{f(x)}$

3. Let $f(x) = \begin{cases} ax+1 & \text{if } x < 2 \\ x^2 & \text{if } x \geq 2 \end{cases}$. Find the value of a for which $\lim_{x \to 2} f(x)$ exists.

4. Use the graph of f to determine whether $\lim_{h \to 0} \dfrac{f(\text{h}) - 2}{h}$ exists. If it does, compute it. If it doesn't, explain how you know.

   Feel free to use either an <u>analytic</u> or a <u>geometric</u> argument to explain your answer. (Hint for geometry: note that $f(0) = 2$ and $f(h) = f(0+h)$; now think *geometrically* about what the limit represents.)

f



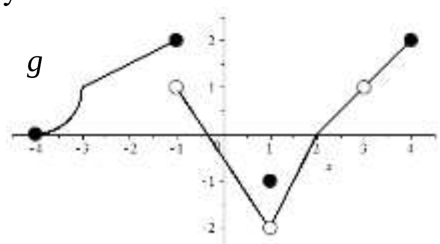
Limits and the derivative: In this section you will need to think about what you learned about the limit definition of the derivative:

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{y \to x} \frac{f(y) - f(x)}{y - x}$$

And then pull that information together with our careful discussion of limits to answer the remaining questions.

5. Use the graph of g to determine whether the limits exist. If a limit exists, compute it. If it doesn't, explain how you know.

g



a. $\lim_{h \to 0} \dfrac{g(-2+h) - \frac{3}{2}}{h}$

b. $\lim_{h \to 0} \dfrac{g(2+h)}{h}$

6. Graph the function $f(x) = |x|$ on the interval [-1,1]. Think carefully as you answer the following questions. (Hint: make of use the graph in setting up the difference quotients in the first two parts of the problem. It will simplify the problem!)

   a. First set up, then evaluate $\lim_{h \to 0^+} \frac{f(0+h) - f(0)}{h}$.

   b. First set up, then evaluate $\lim_{h \to 0^-} \frac{f(0+h) - f(0)}{h}$.

   c. Use this information to show that the function *f* is *not differentiable* at $x = 0$.

   d. Think about zooming in on the graph near zero. What do you see? Is the function locally linear at $x = 0$?

Note: You should see a connection between parts c. and d. of this problem. Do you? If not, you should ask about this!

7. Recall the function that you considered in problem 3: $f(x) = \begin{cases} ax+1 & \text{if } x < 2 \\ x^2 & \text{if } x \geq 2 \end{cases}$. In that problem you found a value of *a* for which $\lim_{x \to 2} f(x)$ exists. If *a* has this value, does $f'(2)$ exist? Use limit definition of the derivative to justify your answer.